Editor's choice

# Basis set convergence of high-order coupled cluster methods up to CCSDTQ567 for a highly multireference molecule 

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## HIGHLIGHTS

- Study post-CCSD(T) contributions to the dissociation energy of $\mathrm{C}_{2}\left({ }^{1} \Sigma_{g}^{+}\right)$.
- Examine contributions up to CCSDTQ567 in conjunction with basis sets up to cc-pV7Z.
- At the CBS limit, post-CCSD(T) contributions add up to $0.427 \mathrm{kcal} \mathrm{mol}^{-1}$.
- Contributions up to CCSDTQ5(6) have to be obtained with relatively large basis sets.
- Contributions beyond CCSDTQ5(6) are smaller than $\sim 2 \mathrm{~cm}^{-1}$.


## A R TICLE INFO

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Basis set extrapolations


#### Abstract

We examine the basis-set convergence of post-CCSD(T) contributions to the dissociation energy of the highly multireference $\mathrm{C}_{2}\left({ }^{1} \Sigma_{g}^{+}\right)$molecule. At the infinite basis set limit we obtain: CCSDT-CCSD(T)/cc-pV\{6,7\} $\mathrm{Z}=-2.268, \operatorname{CCSDT}(\mathrm{Q})-C C S D T / c c-\mathrm{pV}\{6,7\} \mathrm{Z}=3.420, \operatorname{CCSDTQ}-\operatorname{CCSDT}(\mathrm{Q}) / \mathrm{cc}-\mathrm{pV}\{5,6\} \mathrm{Z}=-1.151$, CCSDTQ (5)-CCSDTQ/cc-pV\{Q,5\}Z $=0.412$, CCSDTQ5-CCSDTQ(5)/cc-pV\{T,Q\}Z $=-0.053$, CCSDTQ5(6)-CCSDTQ5/ cc-pV\{D,T\}Z $=0.060, \quad$ CCSDTQ56-CCSDTQ5(6)/cc-pVTZ(1d) $=0.003, \quad$ CCSDTQ56(7)-CCSDTQ56/cc-pVTZ $(1 \mathrm{~d})=0.002$, and CCSDTQ567-CCSDTQ56(7)/cc-pVTZ $(1 \mathrm{~d})=0.001 \mathrm{kcal} / \mathrm{mol}$. These post-CCSD(T) contributions add to $0.427 \mathrm{kcal} / \mathrm{mol}$. Including the $\operatorname{CCSD}(\mathrm{T}) / \mathrm{CBS}$ energy, inner-shell, scalar relativistic, spin-orbit, DBOC, and ZPVE corrections from W4.3 theory results in a CCSDTQ567/CBS $\mathrm{D}_{0}$ value of $144.08 \mathrm{kcal} / \mathrm{mol}$, which agrees to within overlapping uncertainties with the experimental ATcT value of $144.006 \pm 0.06 \mathrm{kcal} / \mathrm{mol}$.


## 1. Introduction

The ground electronic state $\left({ }^{1} \Sigma_{g}^{+}\right)$of the $\mathrm{C}_{2}$ molecule has been extensively investigated both experimentally and theoretically [1-3]. Considerable high-level theoretical work over the past two decades has established $\mathrm{C}_{2}$ as one of the most challenging targets in small molecule computational chemistry [4-38]. This is due to both pathological multireference character in the ${ }^{1} \Sigma_{g}^{+}$state and excessively slow basis set convergence of the higher-order correlation effects. Dicarbon is thus an important and challenging target in benchmarking and development of high-level standard and composite ab initio methods [15,20, 22,26,27,29,32,39-42].

Over a decade ago, a number of high-level studies examined the basis set convergence of the frozen-core $\operatorname{CCSD}(\mathrm{T})$ bond dissociation energy close to the infinite basis set limit using correlation-consistent basis sets as large as aug-cc-pV7Z [20,22,26,27]. These studies have arrived at valence CCSD(T)/ CBS bond dissociation energies ranging between 145.1 and $145.4 \mathrm{kcal} \mathrm{mol}^{-1}$ (where CCSD(T)/CBS stands for coupled cluster with
single, double, and quasiperturbative triple excitations extrapolated to the complete basis set limit). More recently, in a tour de force study, Feller et al. explored the basis set convergence of the frozen-core $\operatorname{CCSD}(\mathrm{T})$ dissociation energy at (or very close to) the infinite basis set limit and arrived at an R/ $\operatorname{UCCSD}(\mathrm{T}) /$ aug-cc-pV\{8,9\}Z value of $145.37 \mathrm{kcal} \mathrm{mol}^{-1}$ [15]. The post$\operatorname{CCSD}(\mathrm{T})$ components of the $\mathrm{C}_{2}$ dissociation energy, on the other hand, have been examined with smaller basis sets. For example, the same study by Feller et al. extrapolated the CCSDT-CCSD(T) and CCSDTQ-CCSDT components from the $c c-p V\{Q, 5\} Z$ and $c c-p V\{T, Q\} Z$ basis set pairs, respectively, whilst CCSDTQ5 contributions were calculated in conjunction with a triple-$\zeta$-quality basis set [15]. In earlier work, [20] which calculated the $\mathrm{C}_{2}$ dissociation energy using W4.3 theory, [29] the post-CCSD(T) components were obtained at the following levels of theories: CCSDT-CCSD(T)/cc-pV \{T,Q\}Z, CCSDT(Q)-CCSDT/cc-pV\{T,Q\}Z, CCSDTQ-CCSDT(Q)/cc-pVTZ, CCSDTQ5/cc-pVDZ, and CCSDTQ5(6)/DZ, where DZ denotes the simple Dunning-Hay [43] valence double- $\zeta$ basis set.

In the present letter, we attempt to converge the post-CCSD(T)

[^0]components of the $\mathrm{C}_{2}\left({ }^{1} \Sigma_{g}^{+}\right)$dissociation energy to the infinite basis set limit. The largest basis sets and basis set extrapolations used are: CCSDT-CCSD(T)/cc-pV\{6,7\}Z, CCSDT(Q)-CCSDT/cc-pV\{6,7\}Z, CCSDTQ-CCSDT(Q)/cc-pV\{5,6\}Z, CCSDTQ(5)-CCSDTQ/cc-pV\{Q,5\}Z, CCSDTQ5-CCSDTQ(5)/cc-pV\{T,Q\}Z, CCSDTQ5(6)-CCSDTQ5/cc-pV \{D,T\}Z, CCSDTQ56-CCSDTQ5(6)/cc-pVTZ(1d), CCSDTQ567/cc-pVTZ (1d), where cc-pVTZ(1d) denotes a truncated version of the cc-pVTZ basis set.

## 2. Computational methods

All calculations were carried out using the MRCC program suite [44,45] with the standard correlation-consistent basis sets of Dunning and co-workers [46-48]. In the post-CCSDTQ5(6) calculations we also use a truncated version of the cc-pVTZ basis set in which the sp functions from the cc-pVTZ basis set are combined with the d functions from the cc-pVDZ basis set (denoted by cc-pVTZ(1d)). The post-CCSD(T) correlation components of the bond dissociation energies are extrapolated using the $\mathrm{E}(L)=\mathrm{E}_{\infty}+\mathrm{A} / L^{3}$ two-point extrapolation formula (where $L$ is the highest angular momentum represented in the basis set). Basis set extrapolations using the cc-pVnZ and cc-pV(n+1)Z basis sets are denoted by cc-pV\{n,n+1\}Z.

All calculations are nonrelativistic and are carried out within the frozen-core approximation, i.e., the 1 s orbitals are constrained to be doubly occupied in all configurations. Thus, for the carbon atom CCSDTQ is equivalent to full configuration interaction (FCI). Coupled cluster calculations for the carbon atom are carried out within the restricted open-shell formalism. The equilibrium bond distance for $\mathrm{C}_{2}\left({ }^{1} \Sigma_{g}^{+}\right)$is taken from Ref. [20]. Finally, we note that some of the higher-order correlation calculations strained our computational resources to the absolute limit. For example, the iterative CCSDTQ/ccp V6Z calculation involves ten billion amplitudes. This calculation required 21 iterations to converge to $10^{-8}$ Hartree. Each iteration ran for about 2.1 days on 20 cores of a dual Intel Xeon machine (E5-2683-v3 and 2.0 GHz ) with 512 GB of RAM, with a total runtime of 46 days. Another big calculation was for the CCSDTQ5/cc-pVQZ energy, which involves six billion amplitudes, and ran for 47 days on an older machine (dual Intel Xeon E5-4650L, $2.60 \mathrm{GHz}, 512 \mathrm{~GB}$ of RAM).

## 3. Results and discussion

In the present letter, we consider iterative and perturbative coupled cluster contributions up to connected septuples (CCSDTQ567). Table 1 gives an overview of the post-CCSD(T) coupled cluster terms that are discussed along with the abbreviations that are used.

Table 2 lists the post- $\operatorname{CCSD}(\mathrm{T})$ correlation components of the $\mathrm{C}_{2}\left({ }^{1} \Sigma_{g}^{+}\right)$dissociation energy. Fig. 1 shows the deviations within $\pm$ $0.15 \mathrm{kcal} \mathrm{mol}^{-1}$ from our best basis set limit values. The T-(T) component converges fairly slowly to the CBS limit. For example, the

Table 1
Overview of the coupled cluster contributions considered in the present work.

| Name | Definition | Abbreviation |
| :--- | :--- | :--- |
| Full-iterative connected triples | CCSDT-CCSD(T) | T-(T) |
| Noniterative connected quadruples | CCSDT(Q)-CCSDT | (Q) |
| Full-iterative connected quadruples | CCSDTQ-CCSDT(Q) | Q-(Q) |
| Connected quadruples as a whole | CCSDTQ-CCSDT | Q |
| Noniterative quintuples | CCSDTQ(5)-CCSDTQ | (5) |
| Full-iterative connected quintuples | CCSDTQ5-CCSDTQ(5) | $5-(5)$ |
| Connected quintuples as a whole | CCSDTQ5-CCSDTQ | 5 |
| Noniterative sextuples | CCSDTQ5(6)-CCSDTQ5 | (6) |
| Full-iterative connected sextuples | CCSDTQ56-CCSDTQ5(6) | $6-(6)$ |
| Connected sextuples as a whole | CCSDTQ56-CCSDTQ5 | 6 |
| Noniterative septuples | CCSDTQ56(7)-CCSDTQ56 | (7) |
| Full-iterative connected septuples | CCSDTQ567-CCSDTQ56(7) | $7-(7)$ |
| Connected septuples as a whole | CCSDTQ567-CCSDTQ56 | 7 |

difference between the cc-pV\{5,6\}Z and cc-pV\{6,7\}Z CBS limits is $0.011 \mathrm{kcal} \mathrm{mol}^{-1}$ (or $3.8 \mathrm{~cm}^{-1}$ ). The cc-pVnZ ( $n=\mathrm{D}, \mathrm{T}, \mathrm{Q}, 5,6,7$ ) basis sets converge smoothly to the CBS limit from above (Fig. 1). In particular, the deviations from the T-(T)/cc-pV\{6,7\}Z limit value are: 1.021 (cc-pVDZ), 0.358 (cc-pVTZ), 0.169 (cc-pVQZ), 0.083 (cc-pV5Z), 0.043 (cc-pV6Z), and 0.027 (cc-pV7Z) kcal mol ${ }^{-1}$. The cc-pV\{D,T\}Z extrapolation achieves the same accuracy as the cc-pV5Z basis set and the cc$\mathrm{pV}\{\mathrm{T}, \mathrm{Q}\} \mathrm{Z}$ extrapolation achieves the same accuracy as the cc-pV7Z basis set.

The ( Q ) component of the dissociation energy converges faster to the CBS limit, such that the difference between the cc-pV\{5, 6$\} \mathrm{Z}$ and cc$\mathrm{pV}\{6,7\} \mathrm{Z}$ CBS limits is merely $0.003 \mathrm{kcal} \mathrm{mol}^{-1}$ (or just $1.0 \mathrm{~cm}^{-1}$ ). The cc-pV\{T,Q\}Z extrapolation results in a deviation of $0.025 \mathrm{kcal} \mathrm{mol}^{-1}$ from the ( Q )/cc-pV\{6,7\}Z energy, whilst the cc-pV\{D,T\}Z extrapolation results in a deviation of $0.035 \mathrm{kcal} \mathrm{mol}^{-1}$ from the ( Q )/cc-pV\{6,7\}Z energy. The cc-pVnZ ( $n=\mathrm{D}, \mathrm{T}, \mathrm{Q}, 5,6,7$ ) basis sets converge smoothly to the CBS limit from below (Fig. 1). In particular, the deviations from the ( Q )/cc-pV\{6,7\}Z limit value are: -0.765 (cc-pVDZ), -0.202 (cc-$\mathrm{pVTZ}),-0.070$ (cc-pVQZ), -0.029 (cc-pV5Z), -0.015 (cc-pV6Z), and -0.010 (cc-pV7Z) $\mathrm{kcal} \mathrm{mol}^{-1}$. The cc-pV\{D,T\}Z extrapolation achieves a similar accuracy as the cc-pV5Z basis set.

For the higher-order quadruples, $\mathrm{Q}-(\mathrm{Q})$, component we were only able to obtain CBS limit values using the cc-pV\{5,6\}Z basis set extrapolation. Nevertheless, since the energy difference between the cc-pV $\{\mathrm{Q}, 5\} \mathrm{Z}$ and $\mathrm{cc}-\mathrm{pV}\{5,6\} \mathrm{Z}$ extrapolations is merely $0.001 \mathrm{kcal} \mathrm{mol}^{-1}$ (or just $0.3 \mathrm{~cm}^{-1}$ ), it is safe to assume that the $\mathrm{Q}-(\mathrm{Q}) / \mathrm{cc}-\mathrm{pV}\{5,6\} \mathrm{Z}$ value is fully converged to the CBS limit. Even the cc-pV\{T,Q\}Z extrapolation is merely $0.003 \mathrm{kcal} \mathrm{mol}^{-1}$ (or $1.0 \mathrm{~cm}^{-1}$ ) away from the cc-pV\{5,6\}Z CBS limit value. However, the cc-pV\{D,T\}Z extrapolation results in a deviation which is one order of magnitude larger, namely $0.033 \mathrm{kcal} \mathrm{mol}^{-1}$. The cc-pVnZ ( $n=\mathrm{D}, \mathrm{T}, \mathrm{Q}, 5,6,7$ ) basis sets converge smoothly to the CBS limit from above. In particular, the deviations from the $\mathrm{Q}-(\mathrm{Q}) / \mathrm{cc}-\mathrm{pV}\{5,6\} \mathrm{Z}$ limit value are: 0.084 (cc-pVDZ), 0.048 (ccpVTZ), 0.022 (cc-pVQZ), 0.011 (cc-pV5Z), and 0.006 (cc-pV6Z) kcal $\mathrm{mol}^{-1}$.

It has been previously found, using smaller basis sets, that there is very effective cancellation between the higher-order triples, CCSDT-CCSD(T), and connected quadruples, CCSDTQ-CCSDT, contributions to the dissociation energy of $\mathrm{C}_{2}\left({ }^{1} \Sigma_{g}^{+}\right)[15,20,26]$. Here we show that at the CBS limit there is perfect cancellation between these two components, namely $\mathrm{T}-(\mathrm{T}) / \mathrm{cc}-\mathrm{pV}\{6,7\} \mathrm{Z}=-2.268 \mathrm{kcal} \mathrm{mol}^{-1}$ and $(\mathrm{Q}) / \mathrm{cc}-\mathrm{pV}\{6,7\} \mathrm{Z}+\mathrm{Q}-(\mathrm{Q}) / \mathrm{cc}-\mathrm{pV}\{5,6\} \mathrm{Z}=+2.269 \mathrm{kcal} \mathrm{mol}^{-1}$.

Our best value for the CCSDTQ(5)-CCSDTQ component is obtained from the cc-pV\{Q,5\}Z extrapolation. As pointed out in much earlier works which obtained this value with the cc-pVDZ basis set $[20,26,29]$ its magnitude is still chemically significant. Specifically, at the (5)/cc$\mathrm{pV}\{\mathrm{Q}, 5\} \mathrm{Z}$ level we obtain a value of $0.412 \mathrm{kcal} \mathrm{mol}^{-1}$ (Table 2). The cc$\mathrm{pV}\{\mathrm{T}, \mathrm{Q}\} \mathrm{Z}$ value of $0.420 \mathrm{kcal} \mathrm{mol}^{-1}$ is only $0.008 \mathrm{kcal} \mathrm{mol}^{-1}$ away from the cc-pV\{Q,5\}Z value (Table 2). Therefore, the cc-pV\{Q,5\}Z value is likely converged to the infinite basis set limit. Nevertheless, the (5)/cc-pV\{D,T\}Z components is clearly not fully converged, being as much as $0.041 \mathrm{kcal} \mathrm{mol}^{-1}$ (or $14 \mathrm{~cm}^{-1}$ ) away from the (5)/cc-pV\{Q,5\} Z value. As opposed to the perturbative, noniterative connected quadruple excitations, which converges fairly smoothly from below to the CBS limit (Fig. 1), the (5)/cc-pVnZ series exhibits oscillatory basis set convergence behaviour with the smaller basis sets. In particular, the deviations from the (5)/cc-pV\{Q,5\}Z CBS limit are: 0.053 (cc-pVDZ), -0.013 (cc-pVTZ), and -0.001 (cc-pVQZ and cc-pV5Z) $\mathrm{kcal} \mathrm{mol}^{-1}$.

We were able to extrapolate the higher-order quintuple excitations, $5-(5)$, from the cc-pV\{T,Q\}Z basis set pair. The $5-(5) / c c-p V\{T, Q\} Z$ value of $-0.053 \mathrm{kcal} \mathrm{mol}^{-1}$ is almost an order of magnitude smaller (in absolute value) than the ( 5 )/cc- $\mathrm{pV}\{\mathrm{Q}, 5\} \mathrm{Z}$ value ( $0.412 \mathrm{kcal} \mathrm{mol}^{-1}$, Table 2). It is hard to estimate how far the $5-(5) / \mathrm{cc}-\mathrm{pV}\{\mathrm{T}, \mathrm{Q}\} \mathrm{Z}$ value is from the true CBS limit, the difference between the $5-(5) / \mathrm{cc}-\mathrm{pV}\{\mathrm{D}, \mathrm{T}\} \mathrm{Z}$ and $5-(5) / \mathrm{cc}-\mathrm{pV}\{\mathrm{T}, \mathrm{Q}\} \mathrm{Z}$ values being $0.027 \mathrm{kcal} \mathrm{mol}^{-1}$. As opposed to the higher-order triple and quadruple excitations (T-(T) and Q-(Q)),

Table 2
Post-CCSD(T) correlation contributions for the $\mathrm{C}_{2}\left({ }^{1} \Sigma_{g}^{+}\right)$dissociation energy calculated (and extrapolated to the CBS limit) with the cc-pVnZ basis sets ( $n=\mathrm{D}, \mathrm{T}, \mathrm{Q}, 5$, 6 , and 7).

| Basis set | T-(T) | (Q) | Q-(Q) | (5) | 5-(5) | (6) | 6-(6) | (7) | 7-(7) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| cc-pVDZ | -1.246 | 2.655 | $-1.067$ | 0.465 | -0.146 | 0.061 | -0.001 | 0.003 | 0.001 |
| cc-pVTZ | -1.910 | 3.218 | -1.103 | 0.399 | -0.062 | 0.060 | $0.003^{\text {a }}$ | $0.002^{\text {a }}$ | $0.001{ }^{\text {a }}$ |
| cc-pVQZ | -2.099 | 3.350 | -1.129 | 0.411 | -0.057 |  |  |  |  |
| cc-pV5Z | -2.185 | 3.391 | -1.140 | 0.411 |  |  |  |  |  |
| cc-pV6Z | -2.224 | 3.405 | -1.145 |  |  |  |  |  |  |
| cc-pV7Z | -2.240 | 3.410 |  |  |  |  |  |  |  |
| cc-pV\{D, T\}Z | -2.190 | 3.456 | -1.118 | 0.371 | -0.026 | 0.060 |  |  |  |
| cc-pV\{T,Q\}Z | -2.237 | 3.445 | -1.148 | 0.420 | -0.053 |  |  |  |  |
| cc-pV\{Q,5\}Z | -2.275 | 3.435 | -1.152 | 0.412 |  |  |  |  |  |
| cc-pV\{5,6\}Z | -2.279 | 3.423 | -1.151 |  |  |  |  |  |  |
| cc-pV\{6,7\}Z | -2.268 | 3.420 |  |  |  |  |  |  |  |

${ }^{\text {a }}$ Truncated version of the cc-pVTZ basis set denoted by cc-pVTZ(1d), see text.


Fig. 1. Basis set truncation errors for the $T-(T)$, (Q), Q-(Q), (5), and 5-(5) components of the $\mathrm{C}_{2}\left({ }^{1} \Sigma_{g}^{+}\right)$dissociation energy calculated in conjunction with the cc-pVnZ basis sets ( $n=\mathrm{D}, \mathrm{T}, \mathrm{Q}, 5,6$, and 7). Reference values are extrapolated from the following basis set pairs: T-(T)/cc-pV\{6,7\}Z, (Q)/cc-pV\{6,7\} Z, Q-(Q)/cc-pV\{5,6\}Z, (5)/cc-pV\{Q,5\}Z, and 5-(5)/cc-pV\{T,Q\}Z.
which converge smoothly from above to the CBS limit (Table 2 and Fig. 1), the $5-(5) / c c-p V n Z$ series converges fairly smoothly to the CBS limit from below. In particular, the deviations from the $5-(5) / \mathrm{cc}-\mathrm{pV}$ \{T,Q\}Z CBS limit value are: -0.093 (cc-pVDZ), -0.009 (cc-pVTZ), and -0.004 (cc-pVQZ) $\mathrm{kcal} \mathrm{mol}^{-1}$.

Our best perturbative, noniterative connected sextuple, CCSDTQ5(6)-CCSDTQ5, component is extrapolated from the cc-pV $\{\mathrm{D}, \mathrm{T}\} \mathrm{Z}$ basis set pair. The (6)/cc-pV\{D,T\}Z value ( $0.060 \mathrm{kcal} \mathrm{mol}^{-1}$ ) is practically the same as the values obtained with the cc-pVTZ and ccpVDZ basis sets, 0.060 and $0.061 \mathrm{kcal} \mathrm{mol}^{-1}$, respectively. Thus it seems like the cc-pV\{D,T\}Z extrapolation is converged to the CBS limit. The (6)/cc-pV\{D,T\}Z component cancels out almost perfectly with our best $5-(5) / \mathrm{cc}-\mathrm{pV}\{\mathrm{T}, \mathrm{Q}\} \mathrm{Z}$ component of $-0.053 \mathrm{kcal} \mathrm{mol}^{-1}$.

We were able to obtain excitations above CCSDTQ5(6) with a truncated version of the cc-pVTZ basis set in which the sp functions from the cc-pVTZ basis set are combined with the d functions from the cc-pVDZ basis set (denoted by cc-pVTZ(1d)). Importantly, all contributions to the binding energy beyond CCSDTQ5(6) are equal to or smaller than $1 \mathrm{~cm}^{-1}$. In particular they are 0.003 (CCSDTQ56-CCSDTQ5(6)), 0.002 (CCSDTQ56(7)-CCSDTQ56), and 0.001 (CCSDTQ567-CCSDTQ56(7)) $\mathrm{kcal} \mathrm{mol}^{-1}$. Overall, the CCSDTQ567-CCSDTQ5(6) difference amounts to $0.006 \mathrm{kcal} \mathrm{mol}^{-1} \approx$ $2 \mathrm{~cm}^{-1}$, thus the $n$-particle limit has been reached at the CCSDTQ5(6) level.

Overall, our best basis set limit post-CCSD(T) contributions to the bond dissociation energy are: $\mathrm{T}-(\mathrm{T}) / \mathrm{cc}-\mathrm{pV}\{6,7\} \mathrm{Z}=-2.268$, $(\mathrm{Q}) / \mathrm{cc}-\mathrm{pV}$ $\{6,7\} \mathrm{Z}=3.420, \quad \mathrm{Q}-(\mathrm{Q}) / \mathrm{cc}-\mathrm{pV}\{5,6\} \mathrm{Z}=-1.151$,
(5)/cc-pV\{Q,5\}
$\mathrm{Z}=0.412, \quad 5-(5) / \mathrm{cc}-\mathrm{pV}\{\mathrm{T}, \mathrm{Q}\} \mathrm{Z}=-0.053, \quad(6) / \mathrm{cc}-\mathrm{pV}\{\mathrm{D}, \mathrm{T}\} \mathrm{Z}=0.060$, $6-(6) / c c-p V T Z(1 d)=0.003,(7) / c c-p V T Z(1 d)=0.002$, and $7-(7) / \mathrm{cc}-$ $\mathrm{pVTZ}(1 \mathrm{~d})=0.001 \mathrm{kcal} \mathrm{mol}^{-1}$. Adding up these post-CCSD(T) corrections gives our best post-CCSD(T) value of $0.427 \mathrm{kcal} \mathrm{mol}^{-1}$. Interestingly, this value is almost spot-on the post-CCSD(T) correction from Feller et al. ${ }^{15}\left(0.44 \mathrm{kcal} \mathrm{mol}^{-1}\right)$ and W4 theory ( $0.42 \mathrm{kcal} \mathrm{mol}^{-1}$ ) [20,29]. Such good agreement is fortuitous and is due to error compensation between the approximated CCSDT-CCSD(T), CCSDTQ, and CCSDTQ5 terms in addition to the complete neglect of the CCSDTQ56 term. For comparison, the post-CCSD(T) correction from W4.3 theory is $0.50 \mathrm{kcal} \mathrm{mol}^{-1}$. Combining our best post-CCSD(T) value of $0.427 \mathrm{kcal} \mathrm{mol}^{-1}$ with the $\operatorname{CCSD}(\mathrm{T}) / \mathrm{CBS}$ energy ( 145.35 ), inner-shell correlation (1.25), scalar relativistic ( -0.17 ), spin-orbit ( -0.17 ), diagonal Born-Oppenheimer ( 0.03 ), and zero-point vibrational energy (2.64) corrections to the bond dissociation energy from W4.3 theory [20,29] results in a relativistic, all-electron, CCSDTQ567/CBS bond dissociation energy at 0 K of $144.08 \mathrm{kcal} \mathrm{mol}^{-1}$. This value agrees to within overlapping uncertainties with the experimental value of $144.006 \pm 0.06 \mathrm{kcal} \mathrm{mol}^{-1}$ from the Active Thermochemical Tables network of Ruscic and co-workers (ATcT, version 1.122e) [49]. We note that using the $\mathrm{R} / \mathrm{UCCSD}(\mathrm{T}) /$ aug-cc-pV\{8,9\}Z energy ( $145.37 \mathrm{kcal} \mathrm{mol}^{-1}$ ) and inner-shell correction ( $1.29 \mathrm{kcal} \mathrm{mol}^{-1}$ ) from ref. [15] instead of the above values from W4.3 theory would further increase the theoretical bond dissociation energy by $0.06 \mathrm{kcal} \mathrm{mol}^{-1}$. The work presented here indicates that our best post-CCSD(T) correction ( $0.427 \mathrm{kcal} \mathrm{mol}^{-1}$ ) is likely to be converged to about $0.01 \mathrm{kcal} \mathrm{mol}^{-1}$, where the main sources of error are from the T-(T)/cc$\mathrm{pV}\{6,7\} \mathrm{Z}$ and $5-(5) / \mathrm{cc}-\mathrm{pV}\{\mathrm{T}, \mathrm{Q}\} \mathrm{Z}$ contributions (vide supra).

## 4. Conclusions

We re-examine the magnitude and basis set convergence of the post$\operatorname{CCSD}(\mathrm{T})$ correlation contributions (up to CCSDTQ567) to the dissociation energy of a pathologically multireference molecule - $\mathrm{C}_{2}\left({ }^{1} \Sigma_{g}^{+}\right)$. At the infinite basis set limit we obtain the following contributions to the dissociation energy: $\mathrm{T}-(\mathrm{T}) / \mathrm{cc}-\mathrm{pV}\{6,7\} \mathrm{Z}=-2.268,(\mathrm{Q}) / \mathrm{cc}-\mathrm{pV}\{6,7\}$ $\mathrm{Z}=3.420, \quad \mathrm{Q}-(\mathrm{Q}) / \mathrm{cc}-\mathrm{pV}\{5,6\} \mathrm{Z}=-1.151, \quad(5) / \mathrm{cc}-\mathrm{pV}\{\mathrm{Q}, 5\} \mathrm{Z}=0.412$, $5-(5) / \mathrm{cc}-\mathrm{pV}\{\mathrm{T}, \mathrm{Q}\} \mathrm{Z}=-0.053, \quad(6) / \mathrm{cc}-\mathrm{pV}\{\mathrm{D}, \mathrm{T}\} \mathrm{Z}=0.060, \quad 6-(6) / \mathrm{cc}-$ $\mathrm{pVTZ}(1 \mathrm{~d})=0.003, \quad(7) / \mathrm{cc}-\mathrm{pVTZ}(1 \mathrm{~d})=0.002$, and $7-(7) / \mathrm{cc}-\mathrm{pVTZ}$ $(1 \mathrm{~d})=0.001 \mathrm{kcal} \mathrm{mol}^{-1}$. Adding up these post-CCSD(T) corrections gives our best post-CCSD(T) value of $0.427 \mathrm{kcal} \mathrm{mol}^{-1}$. Combining this post-CCSD(T) value with the $\operatorname{CCSD}(\mathrm{T}) / \mathrm{CBS}$ energy, inner-shell correlation, scalar relativistic, spin-orbit, diagonal Born-Oppenheimer, and zero-point vibrational energy corrections from W4.3 theory [20,29] results in a relativistic, all-electron, CCSDTQ567/CBS bond dissociation energy at 0 K of $144.08 \mathrm{kcal} \mathrm{mol}^{-1}$, which agrees to within overlapping uncertainties with the experimental ATcT value of $144.006 \pm 0.06 \mathrm{kcal} \mathrm{mol}^{-1}$.

The post-CCSD(T) correlation contributions to the dissociation
energy of this particularly challenging molecule are converged to within $\sim 10 \mathrm{~cm}^{-1}$ using the following basis sets and basis set extrapolations: T-(T)/cc-pV\{T,Q\}Z, (Q)/cc-pV\{T,Q\}Z, Q-(Q)/cc-pV\{D,T\}Z, (5)/cc-pVTZ, 5-(5)/cc-pVTZ, and (6)/cc-pVDZ. The magnitude of higher-order correlation contributions (6-(6), (7), and 7-(7)) is smaller than $\sim 1 \mathrm{~cm}^{-1}$ at the CBS limit. Overall, the CCSDTQ567-CCSDTQ5(6) difference amounts to $0.006 \mathrm{kcal} \mathrm{mol}^{-1} \approx 2 \mathrm{~cm}^{-1}$, thus the $n$-particle limit is practically reached at the CCSDTQ5(6) level.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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